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Abstract: Function is a mathematical term that reflects the relationship between elements of different sets. Rather it is a "law" according to which each element of a set (domain of function) can be put in correspondence with an element of another set (kodomain of function). There is no part of mathematics where mapping is not included, so there is no mathematical theory of this or that way does not include mapping.

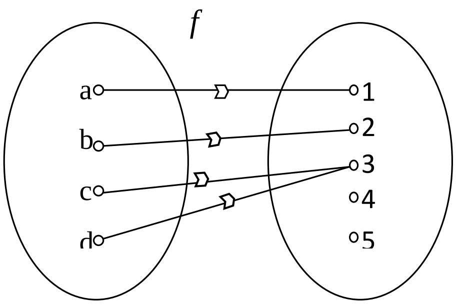
1. Mapping

To define the concept mapping we will depart from other simple terms. Let given two non-empty sets: A and B. Let every element x∈A, is accompanying, a rule f, uniquely determined element y∈B, then we say that is definitely mapping f from A to B and record:

$$\boldsymbol{f}:\boldsymbol{A}\rightarrow\boldsymbol{B}$$

For y we say that is mapping of x and we write

$$\boldsymbol{y = f(x)} \text{ or } \boldsymbol{f: x| \to y} \text{ or } \boldsymbol{x| \to y}$$



Picture 1. Mapping from set A to B.

Set A is named ***domain,*** and the set B is called of ***kodomain*** of the mirroring f.

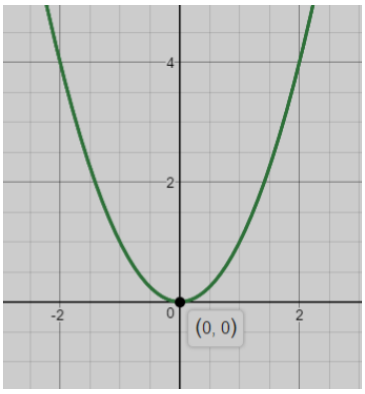
- ***Range*** of the mirrored f or the set of values of the function f, is called set $V_f = f(A)$ which contain all the elements $y \in B$, so $y = f(x)$ for every $x \in D$.
- ***Domain , definition set*** is called set D.
- ***Arguments or independent value*** x is shaped to replace any number of corresponding feature domain.
- With the letter y we indicates the **dependent variable** in the expression $y = f(x)$.

2. Function

Function in mathematics represents the relationship between a group of input values and a group of permitted output values so that each input value is associated with a single output value. For example function that every real number x is associated with its square x^2.That is written as $f(x) = x^2$, if the place of x replace number $x = -3$, we get $f(-3) = 9$, it is obtained for $x = 3$. (Same output value can occur multiple times but one input value **gives only one output value**).

Picture 2.Function f takes value f x gives only an output value

There are many ways a function to be defined or prikazhana. Some functions are defined by the formula or algorithm that tells us how to calculate the output value for a given input value. Another way to show the function is called **graph**.



Слика 3. Graph of the function $f(x) = x^2$

3. Types of Functions

There are many kinds of function. We can be divided into two main groups. **Elementary functions and special functions**, each of which has its subgroups.

– ***Elementary function*** is called each function that can get the basic elementary functions using arithmetic operations (addition, subtraction, multiplication and division) and assembling operation.

  This group includes:

  - Algebraic functions:

- Objectives rational (polynomial) functions;

- Fractional rational functions;

- Irrational functions.

- Transcident functions:

- Exponential functions;

- Logarithmic functions;

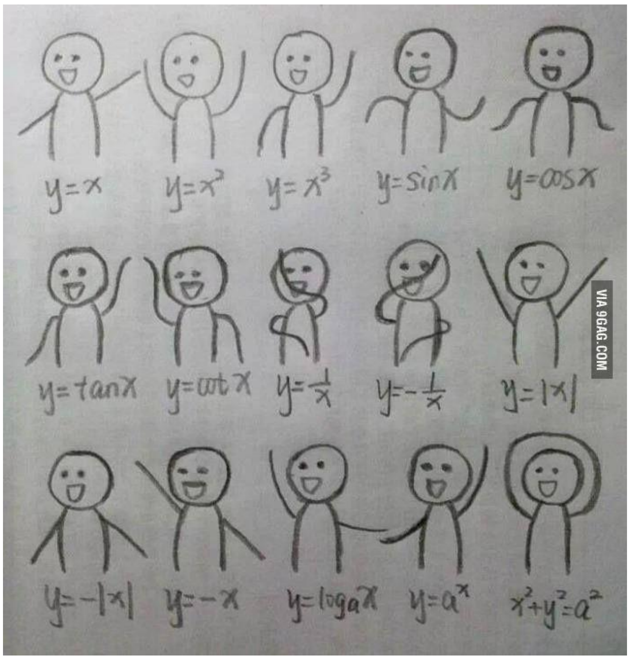
- Trigonometric functions.

- ***Special functions*** - there is no general formal definition, but the list of mathematical functions, there are some that are accepted as special. They are divided into several subgroups:

- Main special features:
- Arithmetic function;
- Conclusion function;
- Elliptic function.

4. Examination properties of Function

Under examination of the function implies a range of different procedures that are performed in order to obtain information about the function such as values that defined function if the function is symmetric, intersection points of a function of coordinate axes, are there asymptotes where rises and falls where, if there extremes saddles. All these tests yields information about the properties of the function on the basis of which we can sketch the graph of the function.



Picture 4.

To draw the graph of a given function $y = f(x)$ we need to examine the following properties:

1. Definition area of the function;
2. Symmetric of the function;
3. Intersections or etc. zeros of the function;
4. Asymptote of the function;
5. Extreme values and intervals monotony of the function;
6. Inflection points, concavity and convexity.

4.1. Definition area

When you say that the function $y = f(x)$) is defined (determined) for a value of $x = y$, it means there is $f(y)$ and that value can be determined. If the function is defined for each value of the interval (a, b), for it is said to be defined that interval. Definition area is determined depending on the analytical expression which is set function. The definition area of a function can be set real numbers or kekoe its subset as interval (open, closed, semi-open and semi-closed), a union of intervals or isolated points.

Computing definition area for some characteristic functions:

– Definition area of polynomial function

  - $x \in R, D_f = (-\infty, +\infty).$

– Definition area of even square root $y = \sqrt[2k]{f(x)}$ $, k \in N$

- Definition area is determined by the inequality $f(x) \geq 0.$

– Definition area of the function quotient $y = \frac{f(x)}{g(x)}$

- The definition area is determined by the the requirement $g(x) \neq 0$ and the definition of functions themselves f и g.

– Definition area of the logarithmic function $y = \ln f(x)$

- The definition area is determined by the inequality $f(x) > 0.$

4.2 **Symmetric of Function**

Under symmetrical properties feature include terms such as **parity, neparnost and periodicity**. Here it investigates and determine whether the function is steam, odd or even steam nor odd (but it is only one of these three cases).

4.3 Even and odd of Function

The function is even if $f(-x) = f(x)$ and graphics function is symmetrical in relation to the y-axis. For odd functions $f(-x) = -f(x)$ and its graph is symmetric with respect to the origin. In the even and odd functions other than having symmetrical schedule also their definition area is symmetric. The third type of functions are those that are neither steam nor odd and among them there is no symmetry in the chart in any definition area.

4.4 Frequency

The function$y = f(x)$, is called periodic with period T is true if the equation $f(x+T) = f(x)$. ПериFrequency function graph indicates that a basic interval of length T is repeated left and right of the said space in the same form as in the basic interval. Since the frequency defenicijata follows that $f(x+kT)$ =f(x), where k is an integer.

Trigonometric functions are periodic functions. The functions $\sin x$ и $\cos x$ are with period $T = 2\pi$, while the functions $\tan x$ и $ctgx$ are with period $T = \pi$.

4.5 Limit points or zeros on a function

The values of the independent variable x for which f (x) = 0 are called zero function. Zeros of the function points of intersection of the function graph with the x-axis. Value at the position y koja argumentit is obtained when x = 0 is the intersection of the y- axis of the graph. Unambiguous feature may have several points of intersection with x- axis, and only one intersection with the y- axis.

4.6 Асимптоти на функцијата

There are three kinds of asymptotes as: vertical, horizontal and oblique and they are determined by the limits. Vertical asymptotes are vertical lines which are the points where the function is defined (there are infinite value). If the function is a rational fraction of the form $y = \frac{f(x)}{g(x)}$, then vertical asymptotes are obtained by solving the equation $g(x) = 0$. The horizontal asymptote is obtained through border $\lim_{x\to\infty} f(x) = a$??? Human then y = a horizontal asymptote. Hair asymptote of form y = kx + n, where $k = \lim_{x\to\infty} \frac{f(x)}{x}$. Function can have one or more vertical asymptotes, the horizontal and hair asymptote mutually exclusive (there can be only one of them).

4.7 Extreme values and the interval monotony

Extreme value

Theorem: If the functions $f(x)$ is diferencial in the interval (a, b) and for some point $x_0 \in (a, b)$ the function have extreme value, then $f'(x_0) = 0$.

The function $f(x)$ has an extreme value $f(x_0)$ if are fulfilled two conditions:

- $f'(x_0) = 0$;
- $f'(x)$ changes sign near x_0.

If the transition to the argument x through point x_0,

- $f'(x)$ changes sign from positive to negative, $y_{max} = f(x_0)$.
- $f'(x)$ changes sign from negative to positive, $y_{min} = f(x_0)$.
- $f'(x)$ does not change sign, then the function will extreme value..

Monotony of the function

It calculate the first derivative of the function and determine the stationary points by solving the equation $f'(x) = 0$. With stationary points is broken deinition area and form intervals of monotony by determining the sign of the first derivative of each of these intervals.

- For the function f (x) which is defined as the interval (a, b) is said to be monotone increasing function of that space if applicable $\forall x_1, x_2 \in (a, b)$ applies:

$$\boldsymbol{x_1 < x_2 \rightarrow f(x_1)} < f(\boldsymbol{x_2})$$

- For the function f (x) which is defined as the interval (a, b) is said to be monotonous opadnuvachka feature that interval if applicable $\forall x_1, x_2 \in (a, b)$ applies:

$$\boldsymbol{x_1 < x_2 \rightarrow f(x_1)} > f(\boldsymbol{x_2})$$

If definition for monotonous growing function of the sign of strict inequality$f(x_1) < f(x_2)$ be replaced with$f(x_1) \leq f(x_2)$, then this function is called monotonous non growing function, and consequently define and monotonous non growing function.

4.8 Inflection points, concavity and convexity

By inflection points are determined intervals of concavity and convexity of the function. The interval at which $f''(x) < x$ the function is convex (concave), and if $f''(x) > x$ function is concave (convex).

The point $\boldsymbol{x} = \boldsymbol{x_0}$ e is **inflection point** of the function ***y=f(x)*** f the curve at that point changes the direction of the bulge (curvature). Home curve that have a fold at the point $\boldsymbol{x} = \boldsymbol{x_0}$ e $\boldsymbol{f''}(\boldsymbol{x_0}) = \boldsymbol{0}$, a sufficient condition is $\boldsymbol{f'''}(\boldsymbol{x_0}) \neq \boldsymbol{0}$.